

Is quantum physics formalizable?



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- ▷ Finding mathematical rigor for a field riddled by **apparent paradoxes** is as old as quantum mechanics itself
- ▷ Entanglement, coherence and measurement¹ pose challenges
- ▷ Are existing **axiomatic systems** sufficient to capture these nuances?
- ▷ **Interactive theorem provers** such as Lean or Isabelle are rapidly maturing
- ▷ Formerly a specialized tool, they can soon become general-purpose **mathematical collaborators**
- ▷ How should proofs be written and verified in the future?²
- ▷ Collaboration with proof-assistants can often lead to a **paradigm shift**
- ▷ Enablers of a new, truly interactive method for developing rigorous results³
- ▷ What are the challenges and benefits of a **hands-on, computer-assisted study** of the foundations of quantum mechanics?

Mathematical foundations



- ▶ « New quantum theory » of the 1920s – 1930s
Heisenberg, Schrödinger, Dirac, Weyl, von Neumann
- ▶ 100 years later, how do we mathematically model ...
 - ... measurement?
 - ... taking the continuum limit to define QFT?
 - ... non-locality and action at a distance?

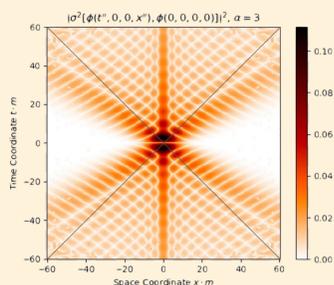
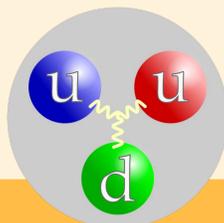


Figure (illustrative). Numerical experiments with a deformed, non-local QFT

- ▶ **Example.** Axioms of Euclidean field theory⁵
 - (E0) Temperedness
 - (E1) Euclidean covariance
 - (E2) Reflection positivity
 - (E3) Symmetry
 - (E4) Cluster property

Axiomatic Yang-Mills



- ▶ Millennium Prize Problem (2000)

Prove that for any compact simple gauge group G , a non-trivial quantum Yang–Mills theory exists on \mathbb{R}^4 and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in Streater & Wightman⁴ and Osterwalder & Schrader.⁵



Computer-verified proofs



Automated Theorem Provers

The computer finds a formal proof, e.g. boolean SAT

$$a^2 + b^2 = c^2$$

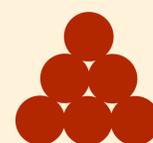


Pythagorean
Triples Problem

Keller's
Conjecture

Interactive Theorem Provers

A proof is manually implemented in a formal language



Kepler's
Conjecture



Four-Color
Theorem



Hilbert's Tenth
Problem

- ▶ Some formalization of physics is being community-run by the *PhysLean* community



References

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4. R. Streater and A. Wightman. *PCT, Spin and Statistics and all That*. W. A. Benjamin, New York, 1964.
5. K. Osterwalder and R. Schrader. Axioms for Euclidean Green's functions, *Comm. Math. Phys.* 31 (1973), 83–112, and *Comm. Math. Phys.* 42 (1975), 281–305.

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