Formalization for (computing) categorified knot invariants

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▷ Khovanov–Lauda–Rouqier–Webster[4] algebras **categorify $U_q(g)$ representation theory** through diagrammatic calculus

▷ Computing the associated invariants in general requires finding projective resolutions

▷ No **general algorithm** is known

▷ Aganagic et al.[1] find that KLRW algebras **arise geometrically** from Fukaya categories (i.e. A_∞-categories)

▷ The geometric picture gives a **procedure-in-principle** to find resolutions

▷ Proof assistants **mechanize reasoning**

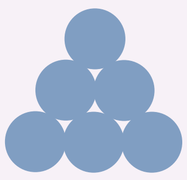
▷ They enable a new, truly **interactive method** for developing rigorous results with the computer[2]

▷ Can formalization help translate the geometric algorithm into **verified and executable programs** and proofs?

LEAN -verified proofs



A history of formally verified proofs.



Kepler's Conjecture

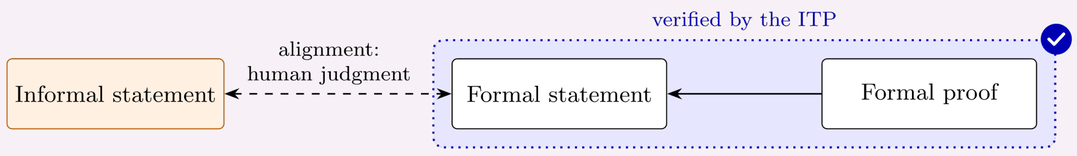


Four-Color Theorem



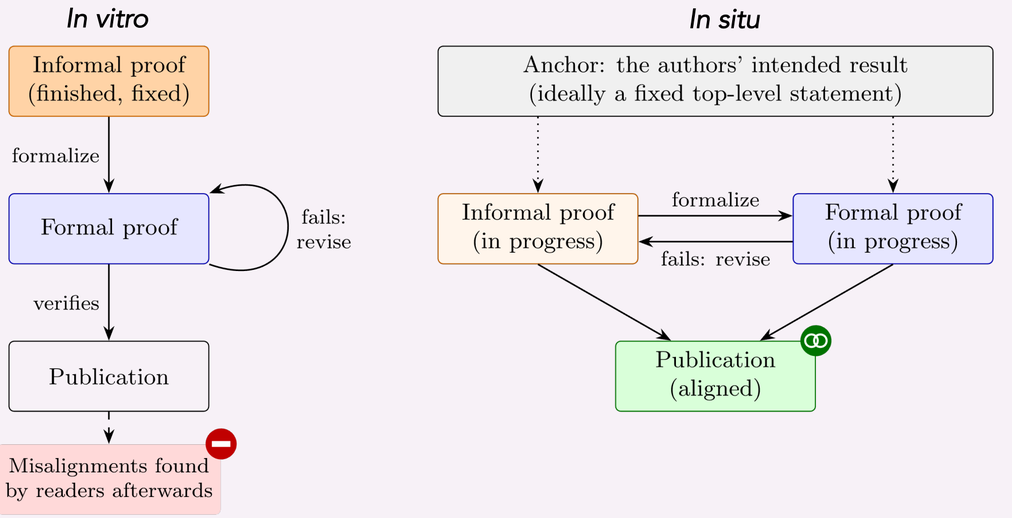
Hilbert's Tenth Problem

- By the Curry–Howard correspondence, we can implement logical reasoning in **type theory**
- Statements become **types**, proofs become **terms**
- **Semantic alignment** of natural-language and formal proofs remain a critical issue:

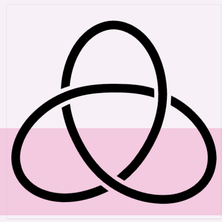


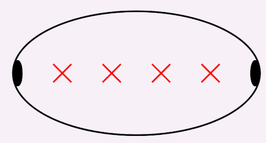
In situ Formalization



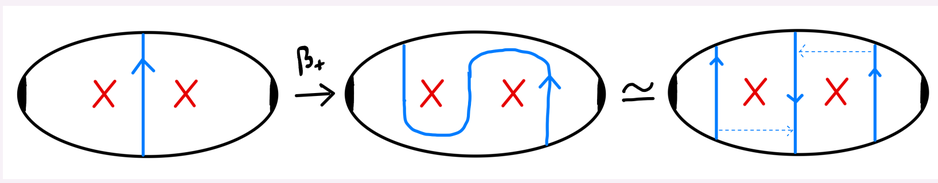


A_∞ braiding functors

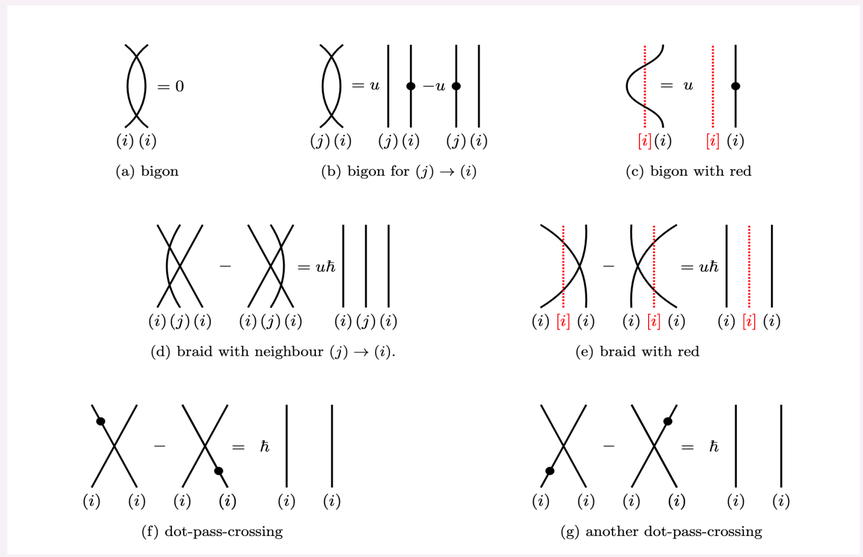


Fuk() ↷ β (braiding)

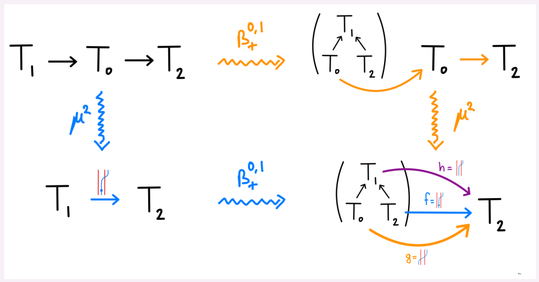
- Braiding acts by a half-twist on the marked points. The resulting objects and morphisms can be read-off after stretching objects to $\pm i\infty$ (producing a resolution)[5]



- After resolution, each objects becomes a chain complex of KLRW-modules[3]



- Acting with braiding on these resolutions requires higher coherences, i.e. higher A_∞ functor components



Example 1.
The A_∞ resolution of the braiding functor acts on the KLRW algebra for sl_2, including higher a higher coherence (purple arrow).

References

1. M. Aganagic, I. Danilenko et al. Quiver Hecke algebras from Floer homology in Coulomb branches. arXiv:2406.04258.
2. J. Bayer, M. David et al. Mathematical proof between generations. *Notices Amer. Math. Soc. (Jan. 2024) Vol. **71**, pp. 79-92.*
3. E. LePage and V. Shende. Aganagic's invariant is Khovanov homology. arXiv:2505.00327.
4. B. Webster. Knot invariants and higher representation theory. *Memoirs Amer. Math. Soc. **250** (2017), no. 1191, pp. 133.*
5. M. Aganagic, E. LePage, M. Rapcak. Homological Link Invariants from Floer Theory. arXiv:2305.13480.

Illustration: Rebecca Wilhelm